JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 103, NO. C2, PAGES 3339-3341, FEBRUARY 15, 1998

Comment on "Energetics of borelike internal waves" by Frank S. Henyey and Antje Hoering

Patrick F. Cummins and Ming Li

Institute of Ocean Sciences, Sidney, British Columbia, Canada

In a recent article appearing in these pages, *Henyey and Hoering* [1997] (hereinafter referred to as H-H) discuss the energetics of undular internal bores in a two-layer fluid. A control volume approach is adopted and one of the main results is the derivation of a relation (their equation (10)) for the total energy flux into an internal bore in the Boussinesq limit, namely,

$$\epsilon^2 = -\frac{\rho U}{2}\left\{HU^2\frac{(h_2-h_1)^2}{h_2^2(H-h_2)^2}\left[Hh_1+h_2(h_2-2h_1)\right] - g'(h_2-h_1)^2\right\}. \qquad (1)$$

The energy flux, ϵ^2, depends on the bore speed, U, the fluid depth, H, and the reduced gravity, $g' = g\Delta\rho/\rho$, with $\Delta\rho$ the density difference between layers and ρ a reference density. Here h_1 and h_2 are the upper layer thicknesses upstream and downstream of the bore, respectively. The quantity ϵ^2 is independent of details within the control volume, in particular the waves and undulations that are often observed superposed on bores. To close the problem, H-H derive a "jump relation," that is, a relation between the bore speed and layer thicknesses.

The main theoretical results of H-H were available in the existing literature, which was largely ignored. The energetics of internal bores for a two-layer fluid were considered by *Long* [1970]. There are slight differences in the assumptions made; *Long* [1970] allows for non-Boussinesq effects and assumes a linear pressure distribution through the jump along a rigid lid, whereas H-H allow for free surface effects to first order. The physical models are otherwise identical. In the Boussinesq limit, equation (23) of *Long* [1970], giving the energy flux into an internal bore, is identical, to within a multiplicative constant (ρ), to equation (10) of H-H.

Jump relations for internal bores have long been a subject of study, but the previous work was not noted by H-H. The derivation of such a relation requires that an assumption be made on the physics of the jump. *Yih and Guha* [1955] derived a relation assuming hydrostatic pressure through the jump. Laboratory experimentation [*Yih and Guha,* 1955; *Long,* 1974; *Baines,* 1984] and numerical modeling [*Cummins,* 1995] have provided empirical support for this jump relation. The closure assumption of *Yih and Guha* [1955] leads to a slight increase of energy across the jump in the contracting layer. Following the work of *Chu and Baddour* [1977], *Wood and Simpson* [1984] propose an alternative relation with an assumption that energy is conserved in one of the layers. On the basis of dye-streak experiments, which indicate that mixing occurs primarily in the expanding layer, *Wood and Simpson* [1984] argue that the energy conservation condition should be applied to the contracting layer. This view has recently been challenged by *Klemp et al.* [1997], who show that assuming energy conservation for the expanding layer is physically plausible and gives better agreement with experiment for large bores advancing into a thin layer. Apparently unaware of these considerations and without justification, H-H also assume energy conservation in the expanding (surface) layer.

Estimates of the energy flux based on (1) are made by H-H for internal bores observed in the Strait of Gibraltar and in Knight Inlet, British Columbia. Field data are used to determine the layer thicknesses and reduced gravities; the chosen jump relation is used to provide one estimate for the bore speed. However, H-H also consider other estimates for the bore speed and show the variation of the total energy flux according to (1) over a wide range of U. This quantity is plotted in Figure 1 for the Strait of Gibraltar example, along with the energy fluxes for the two individual layers. The flux into the surface layer is given by $-\rho U h_1\{gd - 0.5\,U^2[1-(h_1/h_2)^2]\}$, where d is the surface displacement given by equation (8) of H-H (with two sign errors corrected). The energy flux into the deep layer is the difference between the total flux and the flux into the surface layer. The results of Figure 1 show that only for a small range of U are the fluxes positive in both layers. The lower and upper limits of this range are given by the jump relations for energy conservation in the surface and deep layers, respectively. Bore speeds outside the limits imposed by these jump relations imply a potentially large energy gain across the jump for one of the two layers. Since it is commonly assumed that energy is dissipated across the jump in both layers [*Baines,* 1995, section 3.5], it may not be physically acceptable to apply (1) to calculate the energy flux outside the range of speeds where such dissipation is assured.

For bores of moderate amplitude the experimental data reviewed by *Klemp et al.* [1997] suggest that energy is likely to be dissipated in both layers. In this parameter range, which encompasses the observations considered by H-H, the jump relation based on energy

Paper number 97JC02789.
0148-0227/98/97JC-02789$09.00

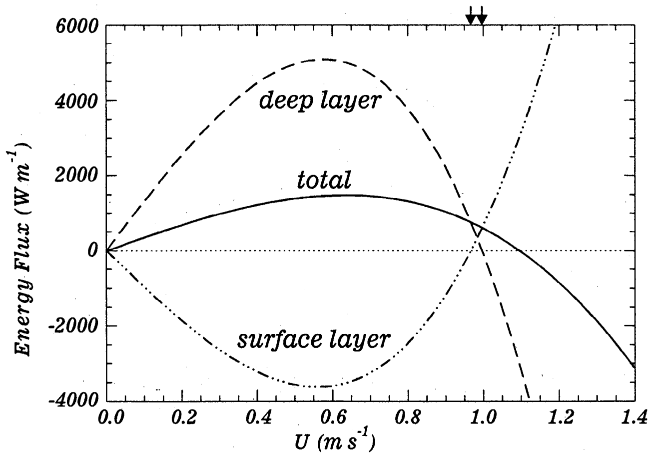

Figure 1. The solid line gives the variation of the total energy flux with bore speed, U, according to (1). Parameter values are $H = 500$ m, $h_1 = 30$ m, $h_2 = 50$ m, $g' = 0.0172$ m s^{-2} and $\rho = 1028$ kg m^{-3}. The dot-dashed and long-dashed lines give energy fluxes into the surface and deep layers, respectively. Positive flux values imply energy losses in the bore. The narrow range of U over which both layers lose energy lies between the two arrows.

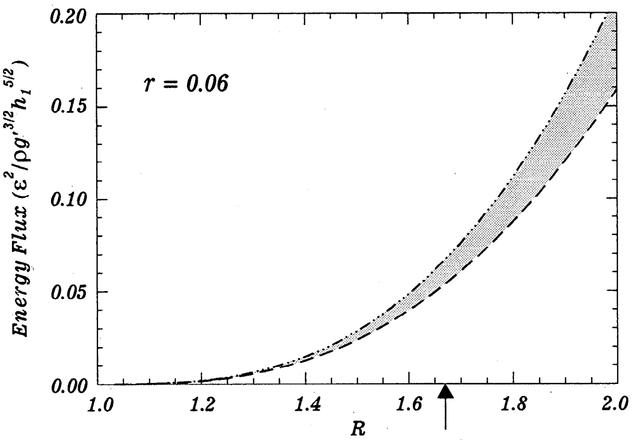

Figure 2. Variation of the nondimensionalized energy flux given by (2) with bore amplitude, R, for $r = 0.06$. The dot-dashed line is for the jump relation (4), with energy conservation in the expanding (surface) layer, whereas the long-dashed curve is for relation (3), assuming conservation in the contracting (deep) layer. The arrow indicates the value of R estimated for the Strait of Gibraltar bore.

conservation in the contracting layer seems to work about as well as one based on conservation in the expanding layer. There is then uncertainty in the choice of an appropriate jump relation, and this may place limits on the accuracy to which the energy flux may be estimated with the two-layer model.

To illustrate this point, (1) is nondimensionalized by letting $U = \sqrt{g'h_1}\, f(R,r)$, where f is the normalized jump relation, $R = h_2/h_1$ is the bore amplitude, and $r = h_1/H$. This yields

$$\frac{\epsilon^2}{\rho g'^{3/2} h_1^{5/2}} = \frac{f(1-R)^2}{2}\left\{1 - \frac{f^2}{R^2}\left[1 + \frac{rR^2(1-r)}{(rR-1)^2}\right]\right\}. \qquad (2)$$

The jump relation for energy conservation in the expanding layer is [*Wood and Simpson,* 1984]

$$f = \left[\frac{R(rR-1)^2(R+1)}{(R^2r - 3Rr + 2)}\right]^{\frac{1}{2}}, \qquad (3)$$

whereas for energy conservation in the contracting layer [*Klemp et al.,* 1997],

$$f = \left[\frac{R^2(rR-1)(rR+r-2)}{(rR^2 - 3rR + R + 1)}\right]^{\frac{1}{2}}. \qquad (4)$$

In Figure 2, nondimensionalized energy fluxes calculated with (3) and (4) are plotted against R, with $r = 0.06$. (Within the parameter range of Figure 2 the relation of *Yih and Guha* [1955] yields results that differ by $< 1\%$ from those obtained with (3).) For the Strait of Gibraltar bore, $R = 1.67$, and the energy flux is about 25% greater with jump relation (4) than with (3). A similar value (32%) is obtained for the Knight Inlet case (not shown). The energy flux is partitioned between the two layers within the shaded region of Figure 2.

For a steadily propagating bore the energy flux is balanced by dissipation. H-H apply a model of dissipation and examine available dissipation data to verify the steady state balance. They find that the energy supply and dissipation agree to within a factor of 2 for the cases considered but conclude that a definitive test of the steady state balance is not possible. The uncertainty introduced to the energy flux calculation due to the jump relations is too small to alter the conclusions of H-H. Such considerations may be significant for more complete data sets when they become available (such as that from a recent experiment in Knight Inlet).

References

Baines, P. G., A unified description of two-layer flow over topography. *J. Fluid Mech., 146,* 127–167, 1984.

Baines, P. G., *Topographic Effects in Stratified Flows,* 482 pp., Cambridge Univ. Press, New York, 1995.

Chu, V. H., and R. E. Baddour, Surges, waves and mixing in two-layer density stratified flow, in *Proceedings of the 17th Congress of the International Association of Hydraulic Research,* vol. 1, pp. 303–310, Baden-Baden, Germany, 1977.

Cummins, P. F., Numerical simulations of upstream bores and solitons in a two-layer flow past an obstacle. *J. Phys. Oceanogr., 25,* 1504–1515, 1995.

Henyey, F. S., and A. Hoering, Energetics of borelike internal waves. *J. Geophys. Res., 102,* 3323–3330, 1997.

Klemp, J. B., R. Rotunno, and W. C. Skamarock, On the propagation of internal bores. *J. Fluid Mech., 331,* 81–106, 1997.

Long, R. R., Blocking effects in flow over obstacles. *Tellus, 22,* 471–480, 1970.

Long, R. R., Some experimental observations of upstream disturbances in a two-fluid system. *Tellus, 26,* 313–317, 1974.

Wood, I. R., and J. E. Simpson, Jumps in layered miscible fluids. *J. Fluid Mech., 140,* 329–342, 1984.

Yih, C. S., and C. R. Guha, Hydraulic jump in a fluid system of two layers. *Tellus, 7,* 358–366, 1955.

P. F. Cummins and M. Li, Institute of Ocean Sciences, P.O. Box 6000, 9860 West Saanich Road, Sidney, BC, Canada V8L 4B2. (e-mail: pcummins@ios.bc.ca)

(Received March 17, 1997; revised June 16, 1997; accepted August 21, 1997.)