



OIL DISPERSION BY TURBULENCE AND COHERENT CIRCULATIONS

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Abstract—Oil spilled over the sea can be broken up into many small oil droplets, which can then be dispersed into the water column by turbulence and coherent circulations, such as Langmuir circulation and thermal convection. The oil dispersion depends critically on the droplet size. Small oil droplets having diameters in the range of a few to hundreds of micrometres are essentially neutrally buoyant particles and they can be dispersed as deep as the coherent circulations can penetrate. Larger oil droplets with diameters of millimetres have buoyant rise speeds comparable with the downwelling velocity of the coherent cells, and they can be suspended in a subsurface retention zone [Stommel (1949), Trajectories of small bodies sinking slowly through convective cells. *J. mar. Res.* 8, 24-29] at the downwelling sites of the cells. By extending the homogenization theory of Rhines and Young [(1983), How rapidly is a passive scalar mixed within closed streamlines? *J. Fluid Mech.* 133, 133-145], we can show that the joint effect of Langmuir circulation and turbulence is to homogenize the oil concentration over the Stommel retention zone. When the buoyant rise speeds of oil droplets are much smaller than the downwelling velocity of the coherent cells, oil concentration will be uniformly distributed across whole cells.

1. INTRODUCTION

FOLLOWING the wreck of the supertanker *BRAER* off the Shetland islands in January 1993, the surface oil slick disappeared from view during the storm following the spill, only to reappear in brown emulsified patches along the west coast of the island when the wind subsided (Pearce, 1993). This curious behaviour suggests that oil can be redistributed below the surface in strong winds, thus greatly complicating the task of tracking and removing it. This phenomenon is not unique to the *BRAER* disaster. Subsurface oil has also been reported in the *ARROW* (Forrester, 1971), the *US/UN POTOMAC* (Petersen, 1978), the *IXTOC-1* blow-out (Payne and Phillips, 1985), the *KURDISTAN* (C-CORE, 1980), the *KATINA* (Rijkwaterstaat, 1982) and the *Thun-tank 5* (OSIR, 1987). The subsurface oil may subsequently rise to the surface or be washed up on coastlines, as happened in the *KATINA* incident and *BRAER* spill.

The density of fresh crude oils varies from 0.75 to 1.0×10^3 kg/m³ (Smedley and Belore, 1991), while seawater's density ranges from 1.0 to 1.03×10^3 kg/m³. Therefore, virtually all crude oil and refined products float when initially spilled. It is thus surprising that oil submerges before the weathering processes can increase the oil density to that of the water. However, the action of water turbulence, including that resulting from breaking waves, can disintegrate floating oil layers into slicklets, blobs or droplets (Bouwmeester and Wallace, 1985; Anon, 1987; Fingas *et al.* 1993). The relatively small oil droplets can then be entrained into the water column by turbulence and subsurface convective motions. It has long been recognized that near surface waters are organized by wind effects into well-defined motions known as Langmuir circulation (Langmuir,

1938). Langmuir circulation consists of counterrotating vortices with their axes aligned with the wind direction. Recent observations (Weller and Price, 1988; Zedel and Farmer, 1991) have shown that downwelling motions beneath the convergence zones of Langmuir cells are very persistent, achieving a downward velocity up to several cm/sec. This vertical velocity is larger than the droplet's rise speed and so can draw oil droplets beneath the surface.

This paper addresses the role of Langmuir circulation in the vertical dispersion of oil droplets. Some mechanical methods used in the oil clean-up operation, such as containment and physical removal, rely upon oil being on or very close to the water surface. These methods would be ineffective if most of the oil has submerged. On the other hand, the process of vertical oil dispersion caused by the addition of chemical dispersants may be accelerated in the presence of Langmuir circulation, so that oil can be more quickly divided into volumes of low concentration that is accessible to biodegradation (Jordan and Payne, 1980).

The nature of the trajectories of oil droplets should not depend necessarily on the nature of the forces producing the cellular motion, so that the results should apply equally well to the thermally induced cellular motion. Thermal convection responsible for the formation of the diurnal thermocline in the ocean surface layer usually occurs during the night, when there is net heat loss at the sea surface.

2. A RANDOM WALK MODEL

A random-walk or Monte-Carlo model is used to study the dispersion of oil droplets, so that changes occurring in individual oil droplets are followed. Provided that the droplet's size is sufficiently small in comparison with the Kolmogorov scale, we can treat oil droplets as point particles. The random-walk model has been successful in studying the dispersion of bubbles in the ocean surface layer (Thorpe, 1984a, b).

Turbulence is simulated by displacement of the water surrounding an oil droplet through a distance L in a random direction δ in (\bar{y}, \bar{z}) -space at each time step $\Delta \bar{t}$. The mixing length L is related to the eddy diffusivity by (see Csanady, 1973)

$$L^2 = 4K_v \Delta \bar{t}. \quad (1)$$

Langmuir circulations are represented by a simple algebraic expression for the stream function

$$\tilde{\psi} = -\frac{w}{k\pi} \sin(k\pi\bar{y}) \sin(l\pi\bar{z}), \quad (2)$$

in which w is the maximum downwelling velocity, $1/k$ and $1/l$ are the width and depth of Langmuir cells (see Fig. 1). Although observations (Weller and Price, 1988) have shown that Langmuir cells are asymmetric cells with the downwelling velocity significantly larger than the upwelling velocity, it is believed that this first order representation provides a useful and relatively straightforward model for estimating oil dispersion.

Superimposing the droplet buoyant rise speed w_b , the velocity induced by Langmuir circulation and the turbulent fluctuation (Thorpe, 1984a), the droplet displacements are

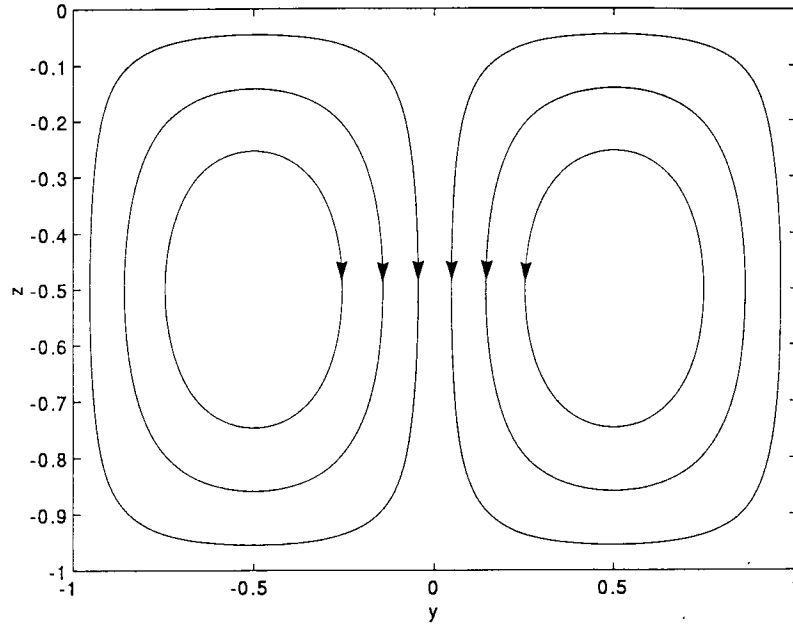


FIG. 1. An illustration of the streamlines of Langmuir cells. The maximum downwelling velocity is w , the width and depth of the cells are $1/k$ and $1/l$, respectively.

$$\Delta \bar{y} = -\frac{wl}{k} \sin(k\pi \bar{y}) \cos(l\pi \bar{z}) \Delta \bar{t} + 2\sqrt{(K_v \Delta \bar{t})} \sin \delta,$$

$$\Delta \bar{z} = [w \cos(k\pi \bar{y}) \sin(l\pi \bar{z}) + w_b] \Delta \bar{t} + 2\sqrt{(K_v \Delta \bar{t})} \cos \delta.$$

Nondimensionalization with respect to length $1/l$ and velocity w leads to

$$\Delta y = -\frac{1}{R_3} \sin(R_3 \pi y) \cos(\pi z) \Delta t + 2\sqrt{(R_1 R_2 \Delta t)} \sin \delta, \quad (3)$$

$$\Delta z = [\cos(R_3 \pi y) \sin(\pi z) + R_2] \Delta t + 2\sqrt{(R_1 R_2 \Delta t)} \cos \delta. \quad (4)$$

Three dimensionless parameters appear in these nondimensionalized equations. $R_1 = K_v l / w_b$ is the ratio of the e -folding depth of the steady-state diffusion to the cell depth. In the absence of Langmuir circulation, the vertical concentration distribution in the steady-state turbulent diffusion is $N \propto \exp\left(\frac{w_b z}{K_v}\right)$. $R_2 = w_b / w$ is the ratio of the droplet rise speed to the maximum downwelling velocity and $R_3 = k/l$ stands for the cell aspect ratio.

Oil droplets are assumed to behave like solid spheres (Leibovich and Lumley, 1982) and their rise speed (Raj, 1977) is given by

$$w_b = \frac{g}{18\nu_w} d^2 (1 - \Delta) \quad (Re < 50) \quad (5)$$

$$= \left[\frac{8g}{3} d(1 - \Delta) \right]^{1/2} \quad (Re > 50) \quad (6)$$

in which g is the gravitational constant, ν_w the kinematic viscosity of water, d the diameter of oil droplets and Δ is the oil to water density ratio. A diagram of droplet rise speed vs diameter is shown in Fig. 2, for typical values of oil:water density ratio. Oil can be broken up into small droplets from a few tens or hundreds of micrometres up to several millimetres in diameter (Forrester, 1971; Smedley and Belore, 1991). Ten-centimetre sized blobs to one-metre sized mats can also be created. For a typical fresh oil—say Arabian Heavy—the oil density is 887 kg/m^3 . When the diameter of oil droplets is in the range of a few to hundreds of micrometres, the rise speed is of order 10^{-7} to 10^{-4} m/sec. For large oil droplets with d in the range of millimetres, the rise speed is a few cm/sec. Weathering processes can increase the oil density (Clark *et al.*, 1987) and hence reduce the droplet rise speed. Evaporation results in the loss of light fractions, leaving the oil more dense. Emulsification—the addition of water to the oil mass in the form of a water-in-oil emulsion—increases the density to a weighted average of the oil and seawater's values. For example, weathered Arabian Heavy can have a density of 951 kg/m^3 . Thus, weathering processes will reduce the oil buoyancy and the buoyant rise speed of weathered oil can be less than half that of fresh oil. Observations of the maximum downwelling velocity of Langmuir circulation lie between

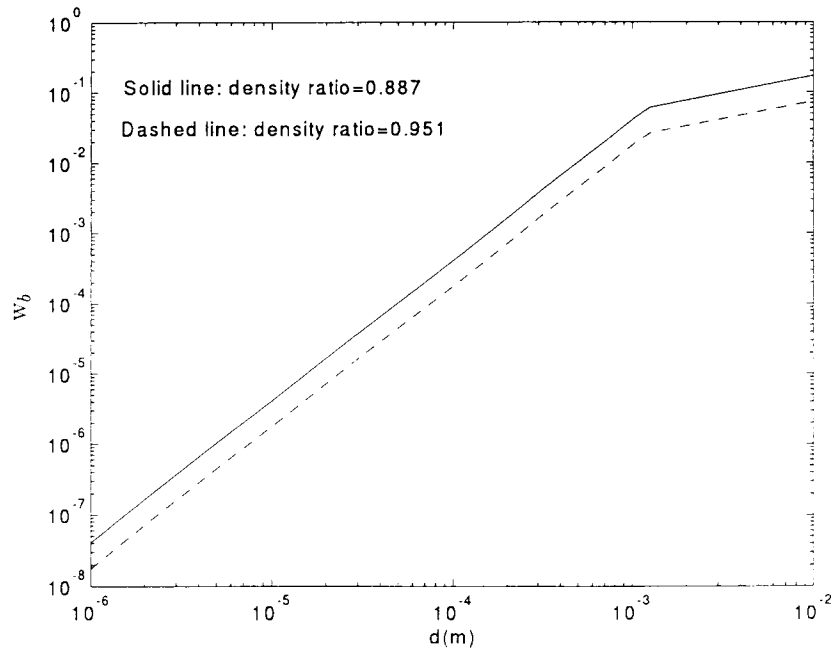


FIG. 2. Oil droplet rise speed vs diameter for freshly spilled and weathered Arabian Heavy oil.

0.05 and 0.3 m/sec. Thus the ratio of droplet rise speed to maximum downwelling velocity R_2 can vary between 10^{-6} and 1.

The eddy diffusivity in the near surface zone is difficult to estimate. Thorpe (1984a) suggested $K_v = 0.018 \text{ m}^2/\text{sec}$ for near surface bubbles fields, which is of similar magnitude to values usually considered appropriate for eddy viscosity in the ocean upper boundary layer. Considering the uncertainty in estimating K_v , we shall in our model explore a wide range, $R_1 = O(10^{-3})$ –10. It will be shown that the value of K_v does not affect the main conclusion of this paper.

Observations by Smith *et al.* (1987) have shown that the maximum spacing between two windrows is about 2–3 times the mixed layer depth and the cell aspect ratio is approximately unity. Zedel and Farmer (1991), using a technique with finer spatial resolution, observed windrow spacings between 5 and 10 m, suggesting that a hierarchy of scales may exist. While recognising the asymmetry as likely to be a feature of near surface circulation, we consider that the simplifying assumption of circular cells ($R_3 = 1$) will nevertheless allow us to represent the essential physics of the process. As better observations of cell structure are observed, the model calculations can easily be adjusted.

3. MODELLING RESULTS

The action of water turbulence, including that generated by breaking waves, can break up an oil film into many small oil droplets and these droplets can be injected into the water in a random fashion (e.g. Fingas, 1993). To represent this injection process, we introduce oil droplets at the upper boundary in random positions $-1 < y < 1$ on the sea surface at each time step, and their positions are followed. Those subsequently reaching $z > 0$ are put back on the surface at $z = 0$, while the droplets reaching $z < -1$ are put back on the bottom boundary $z = -1$. Here we do not consider reflection from the upper and lower boundaries, because neither are solid. Those crossing the vertical boundaries at $y = -1, +1$ are reintroduced at the opposite boundary (see Thorpe, 1984a).

3.1. Homogenization of small oil droplets across Langmuir cells

In this section we study oil droplets which have diameters of tens to hundreds of micrometres. The buoyant rise speed w_b is negligible in comparison with the maximum downwelling velocity w . The dispersion of oil droplets occurs in two stages. First, oil droplets are driven to move along the outermost streamlines of Langmuir cells. Then they are diffused across the streamlines.

Oil droplets randomly introduced at the surface are collected at Langmuir convergence zones, but they cannot submerge without turbulent dispersion, because at the surface the downwelling velocity of Langmuir cells is exactly zero. Once oil droplets are in the water, the downwelling jet in Langmuir cells can push the oil droplets down. Since $R_2 = w_b/w$ is negligible, oil droplets behave like neutrally buoyant particles. At the downwelling site, a vertical oil plume is injected into the water by a downwelling jet, which splits into two horizontal oil plumes when it reaches the lower boundary [see Fig. 3(a)]. These two horizontal jets recirculate back to the surface at the upwelling sites. The thickness of the oil plumes increases with $\sqrt{R_1 R_2} = \sqrt{K_v l/w}$ but is independent of the buoyancy w_b , as is evident from Equations (3) and (4). In other words, the oil

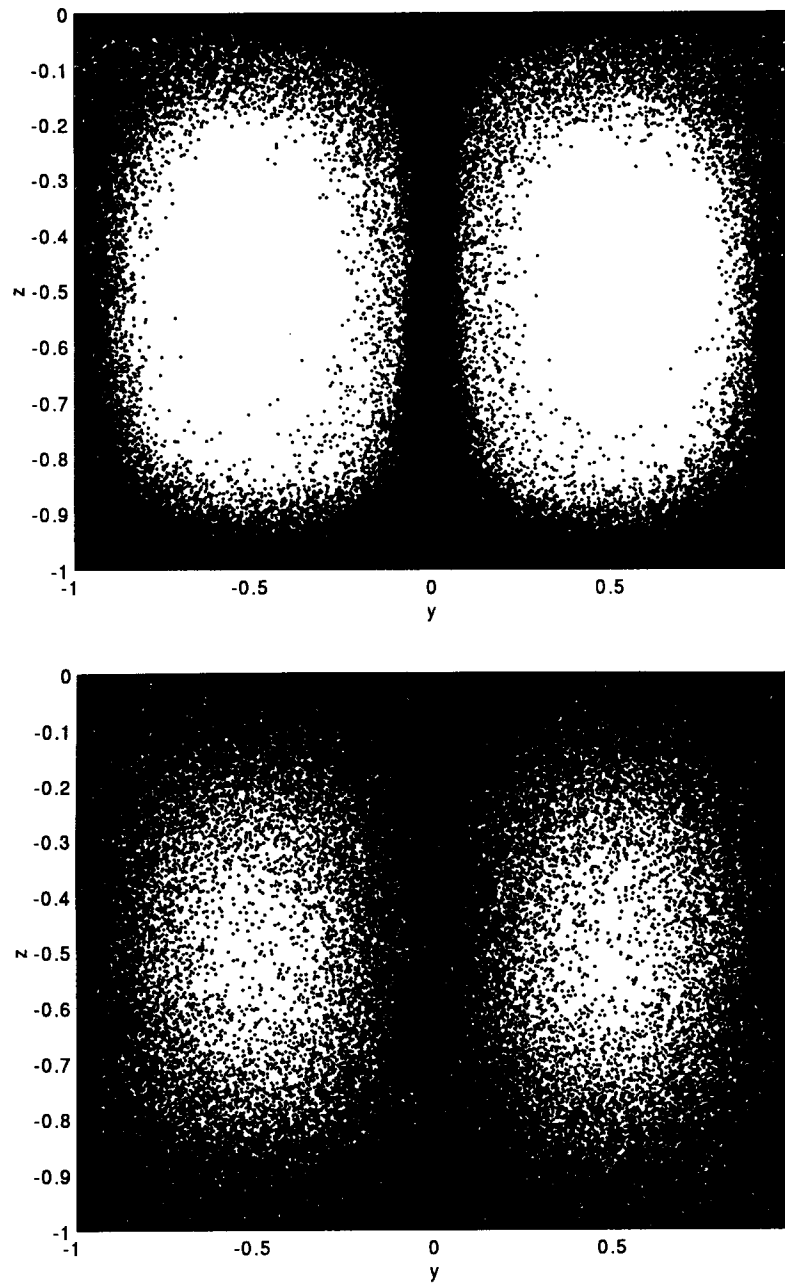


FIG. 3. Dispersion of small oil droplets for $R_1 = 1$, $R_2 = 0.001$, $R_3 = 1.0$. (a) Distribution of oil droplets at $t = 5$. Three-hundred particles are injected at each time step for a total of 500 timesteps. During this advective stage, oil droplets move around the outermost streamlines of Langmuir cells. (b) Distribution of oil droplets at $t = 15$. The total number of oil droplets (or the total volume of oil) is kept fixed for $t > 5$. Oil droplets are diffused across the streamlines.

plumes will be thicker when the turbulent diffusion is stronger, and this thickness increases with time.

In the second stage, the effect of turbulent diffusion dominates. Oil droplets are diffused across streamlines. The oil plumes become progressively thicker with time. Figure 3(b) shows that oil droplets have been diffused across the streamlines of Langmuir cells but are not fully homogenized. There is still a void in the centre of the Langmuir cell, but this void shrinks with time. Eventually, oil concentration will be uniformly distributed throughout the cells. The time taken to achieve uniform distribution is of order $O(1/K_V/k)$ (Rhines and Young, 1983). When the eddy diffusivity K_V is large, homogenization occurs more rapidly. Figure 4 compares the horizontally averaged vertical distribution of oil concentration in the presence or absence of Langmuir circulation. For turbulent diffusion only, without Langmuir circulation, the oil concentration decreases exponentially with depth. In the presence of Langmuir cells, however, there can be significant oil droplet accumulation at greater depths. Thus the existence of organised circulations can have a significant influence on the subsurface distribution of oil.

3.2. Stommel retention zone for large oil droplets

When oil droplets have a diameter of millimetres, they have rise speeds comparable with the maximum downwelling velocity. In this case buoyancy shall act in opposition to the downwelling flow. The buoyant particles descend to depths at which their buoyant rise velocity is equal to the downwards velocity of the flow and so they are

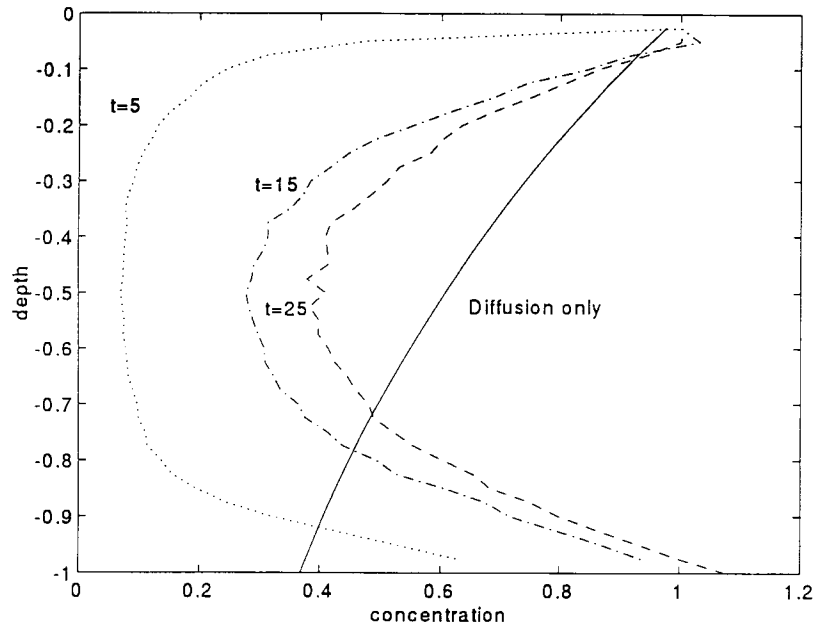


FIG. 4. Oil concentration distribution with depth, averaged across Langmuir cells, at different times. Included for comparison is the exponential profile resulting from turbulent diffusion in the absence of Langmuir circulation.

trapped. This concept was first enunciated by Stommel (1949) and the trapping areas are now known as Stommel retention zones (hereafter SRZ). In this way Langmuir circulation can act as a subsurface trap for oil droplets, lasting for as long as the circulation is sustained, a possibility raised and studied by Leibovich and Lumley (1982).

Leibovich and Lumley (1982) proposed to solve the advection–diffusion equation for oil concentration. Since diffusivity is so small, a straightforward calculation (e.g. by a finite difference algorithm) of the field as a whole was not feasible. Instead they hypothesized that the flow field can be decomposed into four subregions. Analytical results were obtained for a thin boundary-layer region adjacent to the SRZ, but the buffer region between the surface convergence and the SRZ had to be solved numerically.

The random-walk model used in this paper offers an alternative and simpler approach and no hypotheses are required. The advantages of the random-walk model in studying the dispersion of bubbles were well illustrated by Thorpe (1984b). In one numerical run, we have traced the movements of 600 oil droplets for 500 timesteps. Figure 5 shows that a significant number of oil particles have collected at the downwelling site, as well as in the surface layer. In Stommel's model, turbulent diffusion was neglected and consequently the Stommel retention zone is a closed region detached from the surface (Fig. 5). This cannot happen in the presence of turbulence, because turbulent dispersion will maintain the exchange of particles across the boundaries of the retention zone. Our model simulation of droplet distribution generally supports Leibovich and Lumley's conceptual model. Oil droplets collected at the surface convergence are

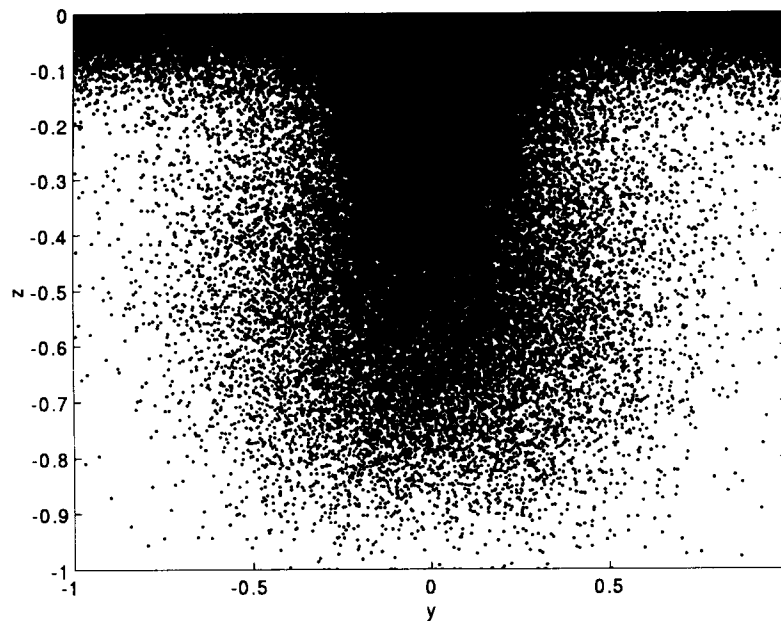


FIG. 5. Formation of a Stommel retention zone for relatively large oil droplets. $R_1 = 0.05$, $R_2 = 0.75$, $R_3 = 1.0$, $T = 5$. Six-hundred particles are injected at each time step for 500 steps.

injected into the SRZ through the buffer region. Outside the central core of the SRZ, there is a broad area where oil droplets are less densely populated. It is thus difficult to locate the boundary-layer region described by Leibovich and Lumley. Furthermore, oil droplets are not only swept into windrows (or convergence zones) but are also diffused downwards, as shown in Fig. 6 by a layer of high oil concentration below the surface.

As demonstrated by Stommel (1949), the retention zone area increases with decreasing R_2 . In physical terms, lighter oil droplets will occupy a larger area.

3.3. A unified view: homogenization of oil concentration over closed streamlines

We have used a Lagrangian model to describe oil dispersion by turbulence and Langmuir circulation. Now we take an Eulerian approach. Thorpe (1984a) gives the steady-state concentration equation

$$v \frac{\partial N}{\partial y} + (w + w_b) \frac{\partial N}{\partial z} = K_v \left(\frac{\partial^2 N}{\partial y^2} + \frac{\partial^2 N}{\partial z^2} \right) \quad (7)$$

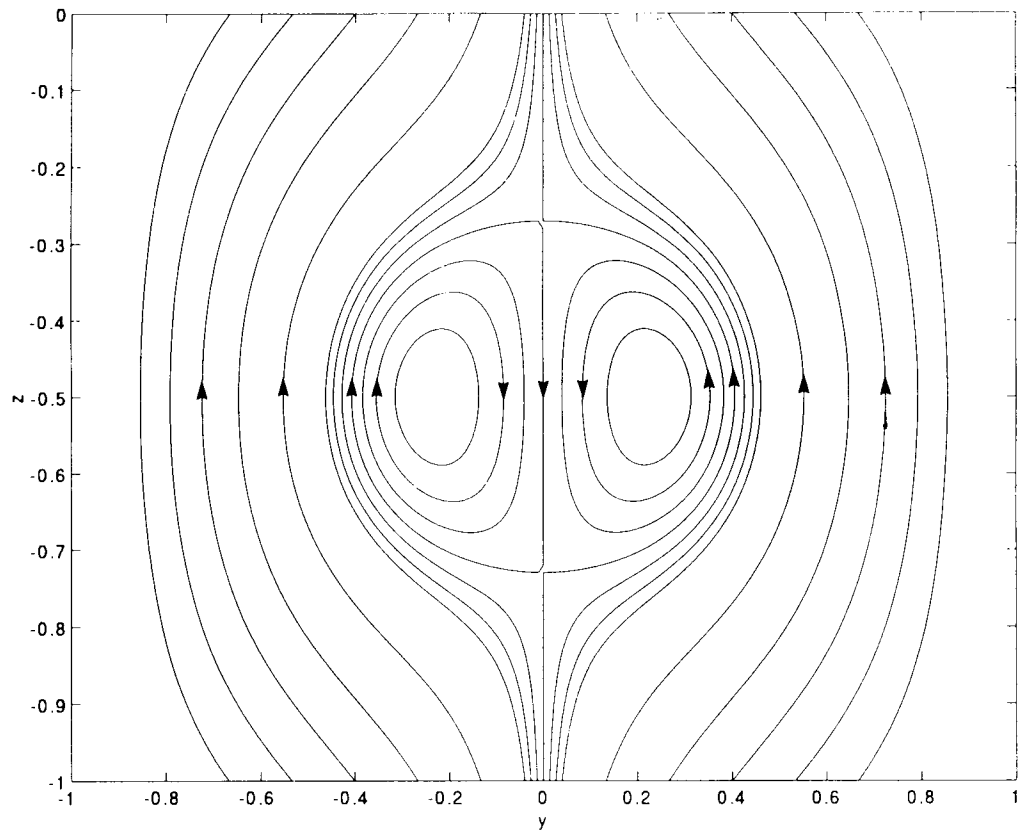


FIG. 6. The subsurface region enclosed by the big circle at the downwelling site of Langmuir circulation is the Stommel retention zone of oil droplets. It consists of two recirculating regions. Buoyant oil droplets descended to this retention zone will remain trapped there in the absence of turbulent diffusion.

where N is the concentration of oil droplets.

Following Batchelor (1956), Rhines and Young (1983) showed that a passive scalar will be homogenized within closed streamlines. Oil droplets having buoyancy are not passive scalars and hence oil concentration will not be homogenized within closed streamlines. Providing there is a sufficient, although small eddy diffusivity, we can demonstrate that the oil concentration N will be homogenized in the Stommel retention zone as the steady state is approached.

Let us define a "modified" stream function ψ_1 such that

$$v = \frac{\partial \psi_1}{\partial z}, \quad w + w_b = -\frac{\partial \psi_1}{\partial y}. \quad (8)$$

It is easy to show that

$$\psi_1 = \psi - w_b y \quad (9)$$

in which ψ is the stream function describing Langmuir circulation.

Defining

$$\frac{dy}{dt} = \frac{\partial \psi_1}{\partial z}, \quad \frac{dz}{dt} = -\frac{\partial \psi_1}{\partial y}, \quad (10)$$

it is apparent that contour lines of ψ_1 are in fact oil droplet trajectories. The region within the closed streamlines of ψ_1 is exactly the Stommel retention zone (see Fig. 6). The SRZ consists of two counter-circulating regions.

In the limit, as K_v goes to zero, Equation (7) may be approximated as

$$J(\psi_1, N) = 0, \quad (11)$$

which leads to

$$N = f(\psi_1). \quad (12)$$

Equation (12) states that N is only a function of ψ_1 .

Now taking an area integral of (7) within any closed contour line of ψ_1 in either half of the SRZ, we obtain

$$\begin{aligned} \iint \nabla^2 N &= \oint \nabla N \cdot n dl \\ &= \frac{dN}{d\psi_1} \oint \mathbf{u} \cdot d\mathbf{l} = 0. \end{aligned} \quad (13)$$

Since $\oint \mathbf{u} \cdot d\mathbf{l}$ is the circulation round the circuit and must be non-zero, we deduce from (13):

$$\frac{dN}{d\psi_1} = 0, \quad (14)$$

that is, N will be homogenized within closed contour lines of ψ_1 . Thus oil concentration will be uniformly distributed in either half of the SRZ.

By symmetry, we conclude that oil concentration will be homogenized within the whole Stommel retention zone. If the oil droplet rise speed is small, as in the case of small droplets, contour lines of ψ_1 will be little different from the streamlines of Langmuir cells. Therefore, small oil droplets will tend to be uniformly distributed throughout the Langmuir cells.

4. DISCUSSION

Depending on their size, oil droplets can be either trapped in a retention zone at the downwelling site or uniformly distributed across the whole Langmuir cell.

For example, small oil droplets with diameter $35\ \mu\text{m}$ and density $887\ \text{kg/m}^3$ have buoyant rise speeds of $w_b = 5 \times 10^{-5}\ \text{m/sec}$. Suppose Langmuir cells have a size of $1/l = 10\ \text{m}$ by $1/k = 10\ \text{m}$ and maximum downwelling velocity of $0.05\ \text{m/sec}$. For an eddy diffusivity of $0.005\ \text{m}^2/\text{sec}$, we find $R_1 = 1$, $R_2 = 0.001$ and $R_3 = 1$. The numerical simulation shown in Section 3.1 shows that it takes 50 min ($t = 15$ in nondimensional time units) to distribute the oil droplets broadly throughout the Langmuir cells.

For oil droplets with diameter $d = 1\ \text{mm}$, the buoyant rise speed is $w_b = 0.0375\ \text{m/sec}$. Suppose the Langmuir cells are the same as before, but we choose a larger eddy diffusivity of $K_v = 0.0188\ \text{m}^2/\text{sec}$ (close to Thorpe's suggestion) giving $R_1 = 0.05$, $R_2 = 0.75$ and $R_3 = 1$. From the numerical simulation of Section 3.2, we find that it takes about 17 min for the oil to collect in the retention zone.

In this model, we have studied oil droplets of a single size but discussed how the dispersion of oil droplets depends on the droplet size. In a real oil spill, oil droplets of different sizes are mixed together. In the absence of oil droplet interaction, large droplets collect in the SRZ, while small droplets are uniformly distributed across the Langmuir cells.

Clark *et al.* (1987) estimated that 30 days of weathering is needed to increase oil density to that of the water. Hence oil submersion caused by weathering takes a long time to realize. Once the surface oil has been broken up into droplets and is injected beneath the surface, its subsequent redistribution by Langmuir circulation can occur in less than 1 hr, lasting as long as the wind is strong enough to sustain the circulation. In the Shetland spill, oil submerged during a storm but resurfaced in calm weather. This strongly suggests that Langmuir circulation, which usually forms when the wind speed exceeds $3\ \text{m/sec}$, can be an important mechanism for oil submersion in the ocean.

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